Math 1552: Integral Calculus
Review Problems for the Midterm Exam
Summer 2021

******NOTE FROM THE INSTRUCTORS: ******
While the list of sections above does not include every integration technique, please note that students are expected to also understand how to integrate with u-substitutions, by parts, and using trig identities, as these techniques may be needed in order to evaluate integrals from the above listed sections. You will find some review problems from previous sections incorporated in the problems below.

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1 Review for Sections: 4.8, 5.1-5.6, 8.2-8.3

Formula Recap

1. Complete each of the following formulas.

(a) The general Riemann Sum is found using the formula:

(b) Some helpful summation formulas are:

$$\sum_{i=1}^{n} c =$$

$$\sum_{i=1}^{n} i =$$

$$\sum_{i=1}^{n} i^2 =$$

(c) Properties of the definite integral:

$$\int_a^a f(x)dx =$$

$$\int_b^a f(x)dx =$$

$$\int_a^b cf(x)dx =$$

(d) State the Fundamental Theorem of Calculus:

(e) Using the FTC:

$$\frac{d}{dx}\left[\int_{a(x)}^{b(x)} f(t)dt\right] =$$

(f) If F is an antiderivative of f, that means:

(g) If F is an antiderivative of f, then:

$$\int f(g(x))g'(x)dx =$$

$$\int_a^b f(g(x))g'(x)dx =$$

(h) To find the area between two curves, use the following steps:

(h) Evaluate an integral using *integration by parts* if:

To choose the value of u, use the rule: ______________. (i) To evaluate integrals with powers or

products of trig functions, use the following trig identities to try to obtain a u-substitution:

2. Fill in the integration formulas below:

$$\int x^n dx, \quad (n \neq -1) =$$

$$\int \sin(ax) dx =$$

$$\int \cos(ax) dx =$$

$$\int \sec^2(ax) dx =$$

$$\int \sec(ax)\tan(ax) dx =$$

$$\int \csc(ax)\cot(ax) dx =$$

$$\int \csc^2(ax) dx =$$

$$\int \frac{1}{1+(ax)^2} dx =$$

$$\int \frac{1}{\sqrt{1-(ax)^2}} dx =$$

$$\int \frac{1}{x} dx =$$

$$\int e^{ax} dx =$$

$$\int b^{ax} dx =$$

$$\int \tan x dx =$$

$$\int \sec x dx =$$

$$\int \csc x dx =$$

$$\int \cot x dx =$$

Problems Similar to the Studio Worksheets

1. True or False?

(a) If F and G are both antiderivatives of f, then $F = G$.

(b) The antiderivative of $\sec^2(3x)$ is $\frac{1}{3}\tan(3x)$.

(c) The indefinite integral of a function f is the collection of all antiderivatives of f.

(d) We know how to find the antiderivative of $\cos(x^2)$, and it is $\sin(x^2)$.

(e) To find the upper sum U_f of a function f on $[a, b]$, after partitioning the interval into n pieces, evaluate f at the right-hand endpoint of each subinterval.

(f) When the interval $[a, b]$ is partitioned into n pieces, there are exactly n endpoints.

(g) A partition of the interval $[a, b]$ does not need to be evenly spaced in order to calculate a Riemann Sum.

(h) If f is positive and continuous on $[a, b]$, and A is the actual area bounded by f, $x = a$, $x = b$, and the x-axis, then $L_f < A < U_f$.

(i) We always set x_i^* to be the right-hand endpoint of the i^{th} interval.

(j) $\sum_{i=1}^{n} i^2 = \left(\frac{n(n+1)}{2}\right)^2$.

(k) If $f(x) \geq 0$ on $[a, b]$, then $\int_a^b f(x)\ dx$ represents the total area bounded by f, $x = a$, $x = b$, and the x-axis.

(l) If f is a continuous function, then the function $F(x) = \int_a^x f(t)dt$ is an anti-derivative of f.

(m) If F is an anti-derivative of f, then $\int_a^b f(x)dx$ represents the slope of the secant line of F(x) on the interval $[a, b]$.

(n) $\frac{d}{dx}\left[\int_a^b f(t)dt\right] = f(x)$.

(o) Given that f is continuous on $[a, b]$ and $F'(x) = f(x)$, then $F(b) - F(a)$ represents the net area bounded by the graph of $y = f(x)$, the lines $x = a$, $x = b$, and the x-axis.

(p) $\int f(x)g(x)\ dx = \left(\int f(x)\ dx\right) \cdot \left(\int g(x)\ dx\right)$

(q) To evaluate $\int \sin^{-1}(x)dx$ by parts, choose $u = \sin^{-1}(x)$ and $dv = dx$.

(r) To evaluate $\int x \ln(x)\, dx$ by parts, choose $u = x$ and $dv = \ln(x)\, dx$.

(s) To evaluate $\int \cot(x)\, dx$, integrate by substitution choosing $u = \sin(x)$.

2. Evaluate the following indefinite integrals.

(a) $\int \left(\sqrt{x} - \frac{1}{x}\right)^2 dx$

(b) $\int \left[4^{-2x} + e^{-5x}\right] dx$

(c) $\int \left(\frac{e^{\sqrt{2}} + x^{\sqrt{2}}}{\sqrt{x}}\right) dx$

(d) $\int \left(\frac{2}{3x} - \frac{1}{\sqrt{4-x^2}}\right) dx$

3. A particle travels with a velocity given by $v(t) = -\frac{1}{3}t^2 + 4t + 2$, where position is measured in meters and time in seconds.

(a) Find the acceleration of the particle when $t = 1$ second.

(b) If the initial position is 4 m, find the position of the particle at $t = 1$ second.

4. (*Applying the Riemann Sum*) You are driving when all of a sudden, you see traffic stopped in front of you. You slam the brakes to come to a stop. While your brakes are applied, the velocity of the car is measured, and you obtain the following measurements: Time since

applying breaks (sec)	0	1	2	3	4	5
Velocity of car (in ft/sec)	88	60	40	25	10	0

(a) Plot the points on a curve of velocity vs. time.

(b) Using the points given, determine upper and lower bounds for the total distance traveled before the car came to a stop.

5. Estimate the area under the graph of $f(x) = 10 - x^2$ between the lines $x = -3$ and $x = 2$ using $n = 5$ equally spaced subintervals, by finding:

(a) the upper sum, U_f.

(b) the lower sum, L_f.

6. (*Applying the Definite Integral*) A marketing company is trying a new campaign. The campaign lasts for three weeks, and during this time, the company finds that it gains customers as a function of time according to the formula:

$$C(t) = 5t - t^2,$$

where t is time in weeks and the number of customers is given in thousands.

Using the general form of the definite integral,

$$\int_a^b f(x)dx = \lim_{n\to\infty} \frac{b-a}{n} \sum_{i=1}^{n} f(x_i^*),$$

calculate the **average** number of customers gained during the three-week campaign.

7. Explain why the following property is true:

$$|\int_a^b f(x)dx| \leq \int_a^b |f(x)|dx.$$

Can you find an example where the inequality is strict? 8. Using the general form of the definite integral, $\int_a^b f(x)dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x$, evaluate:

$$\int_2^4 (x-1)^2 dx.$$

9. Evaluate $\int_0^2 |x-1|dx$ using integral properties from class. (HINT: draw a picture, and use geometry!)

10. Suppose that $f(x)$ is an even function such that $\int_0^2 f(x)dx = 5$ and $\int_0^3 f(x)dx = 8$. Find the value of $\int_{-2}^3 f(x)dx$. 11. Evaluate the integrals:

(a) $\int_1^2 \frac{3x-5}{x^3} dx$.

(b) $\int_2^5 (2-x)(x-5)dx$.

(c) $\int_\pi^{\frac{7\pi}{2}} \frac{1+\cos(2t)}{2} dt$.

12. Find $F'(2)$ for the function

$$F(x) = \int_{\frac{8}{x}}^{x^2} \left(\frac{t}{1-\sqrt{t}}\right) dt.$$

13. (a) Given the function f below, evaluate $\int_1^9 f(x)dx$.

$$f(x) = \begin{Bmatrix} x^2 + 4, & x < 4 \\ \sqrt{x} - x, & x \geq 4 \end{Bmatrix}.$$

(b) Would you get the same answer to part (a) if you evaluated $F(9) - F(1)$? What does this tell you about the FTC and continuity?

14. (a) Evaluate the expressions:

$$\int_0^1 x(1+x)\,dx, \quad \left(\int_0^1 x\,dx \cdot \int_0^1 (1+x)\,dx\right)$$

(b) Looking at your answer in part (a), what, if anything, can you say in general about $\int (f(x) \cdot g(x))dx$?

15. For each integral below, determine if we can evaluate the integral using the method of u-substitution. If the answer is "yes", evaluate the integral.

(a) $\int \frac{1}{x^2} \sec\left(\frac{1}{x}\right) \tan\left(\frac{1}{x}\right) dx$

(b) $\int x \csc^2(x)dx$

(c) $\int \frac{\sin 3x - \cos 3x}{\sin 3x + \cos 3x} dx$

(d) $\int \tan(x^2)dx$

16. Evaluate the following integrals using the method of substitution.

(a) $\int \frac{1}{\ln(x^x)} dx$

(b) $\int \frac{e^{2x}}{\sqrt{4 - 3e^{2x}}} dx$

(c) $\int \frac{dx}{\sqrt{4 - (x+3)^2}}$

17. Suppose that $y = f(x)$ and $y = g(x)$ are both continuous functions on the interval $[a, b]$. Determine if each statement below is always true or sometimes false.

(a) Suppose that $f(c) > g(c)$ for some number $c \in (a, b)$. Then the area bounded by f, g, $x = a$, and $x = b$ can be found by evaluating the integral $\int_a^b (f(x) - g(x))\,dx$.

(b) If $\int_a^b (f(x) - g(x))\, dx$ evaluates to -5, then the area bounded by f, g, $x = a$, and $x = b$ is 5.

(c) If $f(x) > g(x)$ for every $x \in [a, b]$, then $\int_a^b |f(x) - g(x)|\, dx = \int_a^b (f(x) - g(x))\, dx$.

18. Find the area bounded by the region between the curves $f(x) = x^3 + 2x^2$ and $g(x) = x^2 + 2x$.

19. Find the area bounded by the region enclosed by the three curves $y = x^3$, $y = -x$, and $y = -1$.

20. Find the area bounded by the curves $y = \cos x$ and $y = \sin(2x)$ on the interval $\left[0, \frac{\pi}{2}\right]$.

21. Find the area of the triangle with vertices at the points (0,1), (3,4), and (4,2). USE CALCULUS.

22. For each function below: (i) determine which method to use to evaluate the function (formula, u-substitution, or integration by parts, and (ii) evaluate the integral.

(a) $\int_1^e \frac{\sqrt{\ln x}}{x}\, dx$

(b) $\int (\ln x)^2\, dx$

(c) $\int x^2 e^{x^3}\, dx$

(d) $\int x^3 e^{x^2}\, dx$

(e) $\int 4^{-x}\, dx$

(f) $\int x^2 \cdot 4^x\, dx$

23. Determine if each integral below can be evaluated using a method we have learned so far (formula, u-substitution, integration by parts, or trig identities). If so, evaluate the integral. If not, explain why it cannot be evaluated.

(a) $\int x^5 \ln(x)\, dx$

(b) $\int \sin^5(2x) \cos^3(2x)\, dx$

(c) $\int \cos^2(3x)\, dx$

(d) $\int \tan(x) \ln[\cos(x)]\, dx$

(e) $\int \sin\left(x^2\right)\ dx$

(f) $\int \tan^4(x)dx$

(g) $\int e^{2x}\sin(3x)dx$

Additional Midterm Review Problems

24. True or false?

(a) When evaluating a **definite** integral using u-subsitution, different choices of u may lead to different final answers.

(b) Integration by Parts is a Product Rule in integral form.

(c) The goal of integration by parts is to go from an integral $\int f'(x)g'(x)dx$ that we can't evaluate to an integral $\int f(x)g(x)dx$ that we can evaluate.

(d) Definite integrals can not be evaluated by Integration by Parts.

(e) If f is a continuous, increasing function, then the right-hand Riemann sum method always overestimates the definite integral.

(f) Let f be a continuous function and $av(f)$ be the average of f. Then $av(f)\cdot(b-a) = \int_a^b f(x)dx$.

(g) When finding the area between the curves $y = x^3 - x$ and $y = x^2 + x$ it suffices to find the value of the definite integral $\int_{-1}^{2}\left[(x^3-x)-(x^2+x)\right]\ dx$, and then take the absolute value of this value to get the right answer.

(h) To find the area between the curves $y = x^3 - x$ and $y = x^2 + x$, first set the equations equal and solve to find the intersection points $x = -1$ and $x = 2$, plug in a test-point into the equations or graph the curves to determine **top** and **bot**, and then evaluate $\int_{-1}^{2}(\textbf{top}) - (\textbf{bot})\ dx$.

(i) If $\int_0^1 f(x)\ dx = 9$ and $f(x) \geq 0$, then $\int_0^1 \sqrt{f(x)}\ dx = 3$.

25. Evaluate the following integrals.

(a) $\int_0^{\frac{\pi}{4}} \sec^2(t)e^{1+\tan(t)}\ dt$

(b) $\int \sin^3(x)\cos^3(x)\ dx$

(c) $\int \frac{1}{\sqrt{4-9w^2}}\ dw$

(d) $\int x\sin(x)\cos(x)\ dx$

(e) $\int \sec^4(x)\ dx$

(f) $\int \ln(x+1)\ dx$

(g) $\int \left(1+\sqrt{x}\right)^{12} dx$ (HINT: Do not expand out the integrand.)

26. Suppose: $f(1) = 2, f(4) = 7, f'(1) = 5, f'(4) = 3$ and $f''(x)$ is continuous. Find the value of:

$$\int_1^4 xf''(x)\ dx.$$

27. Consider the following limit

$$\lim_{n\to\infty} \sum_{i=1}^{n} \cos\left(\pi \cdot \frac{i}{n}\right) \cdot \frac{\pi}{2n}.$$

(a) Express the limit as a definite integral.

(b) Compute the definite integral from part (a).

28. Let $f(x) = 3x + 4$.

(a) Estimate the area of the region between the graph of f, the lines $x = -1$ and $x = 2$, and the x-axis using a upper sum with three rectangles of equal width.

(b) Find the actual area in part (a) by taking the limit of a general Riemann Sum using n equally spaced subintervals, and taking x_i^* as the right-hand endpoint of each interval.

29. Find the area bounded by the curves $y = \cos^2(x)$ and $y = -\sin^2(x)$, and the lines $x = 0$ and $x = \pi$. (Hint: draw a picture in GeoGebra - an online graphing tool.)

30. Find the area bounded by the curves $y = -x^2 + 6x$ and $y = x^2 - 2x - 24$. (Hint: sketch the curves or make a sign chart.)

31. Find $F'(4)$ if

$$F(x) = \int_{\frac{x^2}{4}}^{x^2} \ln(\sqrt{t})\, dt.$$

32. What value of $b > -1$ maximizes the integral:

$$\int_{-1}^{b} x^2(7-x)dx?$$

33. Find a number c so that $f(c)$ is equal to the average value of the function $f(x) = 1 + x$ on the interval $[-1, 3]$. Graphically, what does that mean?

Answers to Selected Problems

1. (b), (c), (g), (k), (l), (o), (q), (s) are true

2. (a) $\frac{1}{2}x^2 - 4\sqrt{x} - \frac{1}{x} + C$

(b) $-\frac{1}{2\ln 4}4^{-2x} - \frac{1}{5}e^{-5x} + C$

(c) $2e^{\sqrt{2}}\sqrt{x} + \frac{1}{\sqrt{2}+1/2}x^{\sqrt{2}+1/2} + C$

(d) $\frac{2}{3}\ln|x| - \sin^{-1}\left(\frac{x}{2}\right) + C$

3. (a) $\frac{10}{3}\ m/s^2$, (b) $7\frac{8}{9}$ m

4. (b) Upper: 223 ft, Lower: 135 ft

5. (a) 44 (b) 31

6. 4,500 customers

7. Consider the difference between NET and TOTAL area.

8. $\frac{26}{3}$

9. 1

10. 13

11. (a) $-\frac{3}{8}$; (b) $\frac{9}{2}$; (c) $\frac{5\pi}{4}$

12. -24

13. (a) $\frac{79}{6}$; (b) you cannot use the FTC as stated when f is discontinuous somewhere on the interval $[a, b]$

14. (a) $\frac{5}{6}$ and $\frac{3}{4}$; no general rule

15. (a) $-\sec\left(\frac{1}{x}\right) + C$, (c) $-\frac{1}{3}\ln|\sin 3x + \cos 3x| + C$

16. (a) $\ln|\ln x| + C$, (b) $-\frac{1}{3}\sqrt{4 - 3e^{2x}} + C$, (c) $\sin^{-1}\left(\frac{x+3}{2}\right) + C$

17. (c) is true

18. $\frac{37}{12}$

19. $\frac{5}{4}$

20. $\frac{1}{2}$

21. 5. 4.5

22. (a) $\frac{2}{3}$

(b) $x(\ln x)^2 - 2x\ln x + 2x + C$

(c) $\frac{1}{3}e^{x^3} + C$

(d) $\frac{x^2 e^{x^2}}{2} - \frac{e^{x^2}}{2} + C$

(e) $-\frac{1}{\ln 4}4^{-x} + C$

(f) $\frac{1}{\ln 4}x^2 \cdot 4^x - \frac{2}{(\ln 4)^2}x \cdot 4^x + \frac{2}{(\ln 4)^3}4^x + C$

23. (a) $\frac{x^6 \ln x}{6} - \frac{x^6}{36} + C$

(b) $\frac{1}{12}\sin^6(2x) - \frac{1}{16}\sin^8(2x) + C$

(c) $\frac{1}{2}x + \frac{1}{12}\sin(6x) + C$

(d) $-\frac{1}{2}(\ln[\cos(x)])^2 + C$

(e) Cannot be evaluated

(f) $\frac{1}{3}\tan^3(x) - \tan(x) + x + C$

(g) $\frac{2}{13}e^{2x}\sin(3x) - \frac{3}{13}e^{2x}\cos(3x) + C$

24. (e), (f) are true

25. (a) $e^2 - e$

(b) $\frac{1}{6}\cos^6(x) - \frac{1}{4}\cos^4(x) + C$

(c) $\frac{1}{3}\arcsin\left(\frac{3w}{2}\right) + C$

(d) $\frac{x}{2}\sin^2 x - \frac{1}{4}x + \frac{1}{8}\sin 2x + C$

(e) $\tan(x) + \frac{\tan^3(x)}{3} + C$

(f) $(x+1)\ln(x+1) - (x+1) + C$

(g) $\frac{1}{7}(1+\sqrt{x})^{14} - \frac{2}{13}(1+\sqrt{x})^{13} + C$

26. 2

27. (a) $\int_0^{\frac{\pi}{2}} \cos(2x)dx$, (b) 0

28. (a) 21, (b) 16.5

29. π

30. $\frac{512}{3}$ or approximately 170.67

31. $14\ln 2$

32. $b = 7$

33. $c = 1$

2 Review for Sections: 4.5, 8.4-8.5, 8.8

Content Recap

(a) To apply L'Hopital's rule, the limit must have the indeterminate form _____________ or ______________.

(b) An integral $\int_a^b f(x)dx$ is *improper* if at least one of the limits of integration is _____________, or if there is a _____________ _____________ on the interval $[a, b]$.

(c) If we would evaluate an integral using *trig substitution*, the integral should contain an expression of one of these forms: _____________, _____________, or _____________.

Write out the trig substitution you would use for each form listed above.

(d) To use the method of *partial fractions*, we must first factor the denominator completely into _____________ or _____________ _____________ terms.

In the partial fraction decomposition, if the term in the denominator is raised to the kth power, then we have _____________ partial fractions.

For each linear term, the numerator of the partial fraction will be _____________.

For each irreducible quadratic term, the numerator will be _____________.

Studio Worksheet Type Problems

2. Determine if the following statements below are always true or sometimes false.

(a) If an integral contains the term $a^2 + x^2$, we should use the substitution $x = a \sec\theta$.

(b) The expression $\tan\left(\sin^{-1}(x)\right)$ cannot be simplified.

(c) When using a trig substitution with a term of the form $a^2 - x^2$, we could use either $x = a\sin\theta$ or $x = a\cos\theta$ and obtain equivalent answers (that may differ only by a constant).

(d) If we use the trig substitution $x = \sin\theta$, then it is possible that $\sqrt{1 - x^2} = -\cos\theta$.

(e) The partial fraction decomposition of $\frac{x}{(x+3)^2}$ is $\frac{A}{x+3} + \frac{B}{(x+3)^2}$.

(f) $\int \frac{dx}{(x+3)^2} = \ln(x+3)^2 + C$.

(g) The integral $\int \frac{x}{x^2-9}dx$ could be best evaluated using the method of partial fractions.

(h) The integral $\int \frac{dx}{x(x^4+1)}$ cannot be evaluated using the method of partial fractions.

(i) $\lim_{x\to\infty} xe^x$ has the indeterminate form ∞^∞.

(j) $\lim_{x\to 0^+}(\cos x)^{\frac{1}{x}}$ has the indeterminate form 1^∞.

(k) $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{2x} = 2e$.

(l) When evaluating a limit using L'Hopital's rule, we first need to find $\left(\frac{f}{g}\right)'$.

(m) If f has a vertical asymptote at $x=a$, then $\int_a^b f(x)dx = \lim_{c\to a^+}\int_c^b f(x)dx$.

(n) $\int_{-1}^{1}\frac{1}{x}dx = 0$.

(o) Saying that an improper integral converges means that the integral must evaluate to a finite number.

(p) Indefinite integrals can be improper.

3. Evaluate the following integrals using any method we have learned so far: u-substitutions, integration by parts, integrating trig functions, trigonometric substitutions, or partial fractions.

(a) $\int \frac{x^2}{(x^2+4)^{3/2}}dx$

(b) $\int (x^2+1)e^{2x}dx$

(c) $\int \frac{\sqrt{1-x^2}}{x^4}dx$

(d) $\int \frac{dx}{e^x\sqrt{e^{2x}-9}}$

(e) $\int \sin^2(x)\cos^2(x)dx$

(f) $\int \frac{x+3}{(x-1)(x^2-4x+4)}dx$

(g) $\int x^5\ln(x)dx$

(h) $\int \frac{x+4}{x^3+x}dx$

(i) $\int \sqrt{25-x^2}dx$

(j) $\int \frac{x-1}{(x+1)^3} dx$

(k) $\int \frac{x+2}{x+1} dx$

(l) $\int \frac{x+1}{x^2(x-1)} dx$

4. Evaluate the following limits using L'Hopital's Rule.

(a) $\lim_{x\to 0^+}[x(\ln(x))^2]$

(b) $\lim_{x\to\infty}(x+e^x)^{2/x}$

(c) $\lim_{x\to \frac{\pi}{2}}\left[\frac{\ln(\sin x)}{(\pi-2x)^2}\right]$

5. Evaluate the improper integrals if they converge, or show that the integral diverges.

(a) $\int_0^3 \frac{x}{(x^2-1)^{2/3}} dx$

(b) $\int_0^\infty x^2 e^{-2x} dx$

(c) $\int_1^4 \frac{dx}{x^2-5x+6}$

Additional Midterm Review Problems

10. Determine if each statement below is always true or sometimes false.

(a) $\lim_{x\to 2} \frac{x-2}{x^2+x-6}$ is of an indeterminate form.

(b) $\lim_{x\to\infty}\left(1-\frac{1}{x}\right)^x = e$

(c) The integral $\int x^3\sqrt{1-x^2}\, dx$ can be evaluated by trigonometric substitution by setting $x = \sin x$.

(d) $\sin(\cos^{-1}(x)) = \tan(x)$.

(e) For the rational expression $\frac{x}{(x+10)(x-10)^2}$, the partial fraction decomposition is of the form $\frac{A}{x+10} + \frac{B}{(x-10)^2}$.

(f) For the rational expression $\frac{2x+3}{x^2(x+2)^2}$, the partial fraction decomposition is of the form $\frac{A}{x^2} + \frac{B}{x+2} + \frac{C}{(x+2)^2}$.

(m) The integral $\int_{-1}^{1} \frac{1}{x^2}\, dx$ can be evaluated using the Fundamental Theorem of Calculus.

11. Evaluate each integral using any method we have learned.

(a) $\int \frac{2x+1}{x^2-7x+12}\, dx$

(b) $\int \frac{8\, dx}{x^2\sqrt{4-x^2}}$

(c) $\int \frac{8\, dx}{(4x^2+1)^2}$

(d) $\int \frac{1}{(x+1)(x^2+1)}\, dx$

12. Use L'Hopital's rule to evaluate the following limits.

(a) $\lim_{x\to\infty} (\ln x)^{\frac{1}{x^2+1}}$

(b) $\lim_{x\to 0^+} (\ln x)^x$

(c) $\lim_{x\to 0} \left[\frac{1}{x} - \cot x\right]$

(d) $\lim_{x\to\infty} \left[\cos\left(\frac{1}{x}\right)\right]^x$

13. Find values of a and b so that

$$\lim_{x\to 0} \frac{\cos(ax) - b}{2x^2} = -4.$$

Answers to Selected Questions

2. (c), (e), (h), (j), (m), (o), (s), (u), (w), (x), (y) are true

3. (a) $\ln\left|\frac{\sqrt{x^2+4}}{2} + \frac{x}{2}\right| - \frac{x}{\sqrt{x^2+4}} + C$ (b) $\frac{1}{2}(x^2+1)e^{2x} - \frac{1}{2}xe^{2x} + \frac{1}{4}e^{2x} + C$

(c) $-\frac{1}{3} \cdot \frac{(1-x^2)^{3/2}}{x^3} + C$ (d) $\frac{\sqrt{e^{2x}-9}}{9e^x} + C$ (e) $\frac{x}{8} - \frac{1}{32}\sin(4x) + C$

(f) $4\ln\left|\frac{x-1}{x-2}\right| - \frac{5}{x-2} + C$ (partial fractions)

(g) $\frac{x^6 \ln x}{6} - \frac{x^6}{36} + C$ (by parts)

(h) $4\ln|x| - 2\ln(x^2+1) + \tan^{-1}(x) + C$ (partial fractions)

(i) $\frac{25}{2}\sin^{-1}\left(\frac{x}{5}\right) + \frac{x\sqrt{25-x^2}}{2} + C$ (trig sub)

(j) $-\frac{1}{x+1} + \frac{1}{(x+1)^2} + C$ (k) $x + \ln|x+1| + C$

(l) $-2\ln|x| + \frac{1}{x} + 2\ln|x-1| + C$

4. (a) 0, (b) e^2, (c) $-\frac{1}{8}$

5. (a) $\frac{9}{2}$, (b) $\frac{1}{4}$, (c) diverges

10. (a), (c), (i), (j), (l) are true

11. (a) $-7\ln|x-3| + 9\ln|x-4| + C$, (b) $\frac{-2\sqrt{4-x^2}}{x} + C$

(c) $2\tan^{-1}(2x) + \frac{4x}{4x^2+1} + C$, (d) $\frac{1}{2}\ln|x+1| + \frac{1}{2}\arctan x - \frac{1}{4}\ln|x^2+1| + C$

12. (a) 1, (b) 1, (c) 0, (d) 1

13. $a = \pm 4$, $b = 1$